# DIOPHANTINE ANALYSIS.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

## SOLUTIONS TO PROBLEMS.

**3. Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Logan County, Ohio.**

It is required to find three whole numbers in an arithmetical progression, such that the sum of every two of them shall be a square.

**II. Solution by ARTEMAS MARTIN, A. M., Ph.D., LL.D., U. S. Coast and Geodetic Survey Office, Washington, D. C.**

Let $x-y$, $x$ and $x+y$ denote the required numbers in arithmetical progression. Then must

$2x-y=\square=a^2\ldots(1)$, $2x=\square=b^2\ldots(2)$, $2x+y=\square=c^2\ldots(3)$.

Substituting $x=\frac{1}{2}b^2$, the value given by (2), in (1) and (3) we get

$b^2-y=\square=a^2\ \ldots(4)$, and $b^2+y=\square=c^2\ldots(5)$.

From (4) and (5) we find

$y=b^2-a^2=c^2-b^2\ldots(6)$, therefore $2b^2=a^2+c^2\ldots(7)$,

which is the only condition remaining to be satisfied.

Let $c=m+n$, and $a=m-n$, then (7) becomes

$b^2=m^2+n^2\ldots(8)$,

which is satisfied by assuming $m=2pq$, $n=p^2-q^2$,

Hence $x=\frac{1}{2}b^2=\frac{1}{2}(m^2+n^2)=\frac{1}{2}(p^2+q^2)^2$,

$y=b^2-a^2=(m^2+n^2)-(m-n)^2=2mn=4pq(p^2-q^2)$,

and the required numbers are

$x-y=\frac{1}{2}(m^2+n^2)-2mn=\frac{1}{2}(p^2+q^2)^2,-4pq(p^2-q^2)$,

$x=\frac{1}{2}(m^2+n^2)=\frac{1}{2}(p^2+q^2)^2$,

$x+y=\frac{1}{2}(m^2+n^2)+2mn=\frac{1}{2}(p^2+q^2)^2+4pq(p^2-q^2)$.

Taking $p=5$, $q=4$, we get $x=840\frac{1}{2}$, $y=720$; hence $x-y=120\frac{1}{2}$, $x+y=1560\frac{1}{2}$, and multiplying by 4 for integers the required numbers are found to be 482, 3362 and 6242.

This set of numbers is the same as that found by different methods of solution in Maynard's edition of the key to Bonnycastle's Introduction to Algebra, published in London in 1835. See pp. 113-115.

An infinite number of other sets may be found.

**4. Proposed by H. W. HOLYCROSS, Superintendent of Schools, Pottersburg, Union Co., Ohio.**

What value of $x$ will render $4x^4+12x^3-3x^2-2x+1$ a square?

**II. Solution by ARTEMAS MARTIN, A.M., Ph.D., LL.D., U. S. Coast and Geodetic Survey Office, Washington, D. C.**

Put $4x^4+12x^3-3x^2-2x+1=(2x^2+3x-3)^2$,

$=4x^4+12x^3-3x^2-18x+9$;

whence $x=\frac{1}{2}$. *Other values may be found.*

[Dr. Martin also sent excellent solutions to Nos. 1 and 2. R. H. Young, of West Sunbury, Pa., and Alvin E. Schmidt, Winesburg, Ohio, sent solutions to 1, 3

and 4. These solutions were not received in time to be acknowledged in March No.—ED.]

## PROBLEMS.

9. **Proposed by ISAAC I. BEVERAGE, Monterey, Virginia.**

It is required to find three numbers the sum of whose 4th power is a square.

10. **Proposed by L. B. HAYWARD, Bingham, Ohio.**

Find two numbers such that each of them and also their sum and their difference when increased by unity shall all be square numbers.

# AVERAGE AND PROBABILITY.

**Conducted by B. F. FINKEL, Kidder, Mo. All contributions to this department should be sent to him.**

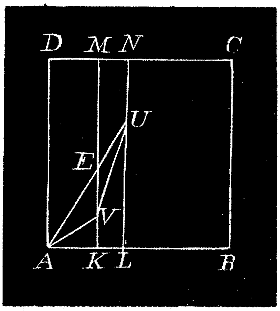
## SOLUTIONS TO PROBLEMS.

2. **Proposed by O. S. KIBLER, Superintendent of Schools, West Middleburg, Logan Co., Ohio.**

Find the average area of a triangle formed by joining an angle of a square with any two points within the square.

**Solution by Professor G. B. M. ZERR, Principal of High School, Staunton, Virginia.**

Let $ABCD$ be the square side $a$, and $U$, $V$, the two random points.




Through $V$, $U$ draw $KM$, $NL$, parallel to $AD$, $KM$ meeting $AU$ in $E$.

Let $AL=X$, $AK=w$, $LU=y$, $KV=z$, $KE=z'$.

Then $z'=\dfrac{wy}{x}$; also

area $AUV=\frac{1}{2}(wy-xz)=u$, when $z<z'$,
area $AUV=\frac{1}{2}(xz-wy)=u_1$, when $z>z'$.

The limits of $x$ are 0 and $a$; of $w$, 0 and $x$; of $y$, 0 and $a$; of $z$, 0 and $z'$, and $z'$ and $a$.

Hence, the required average area is

$$\Delta=\frac{\int_0^a\int_0^x\int_0^a\left\{\int_0^{z'} udz+\int_{z'}^a u_1dz\right\}dxdwdy}{\int_0^a\int_0^x\int_0^a\int_0^a dxdwdydz.}$$

$$=\frac{2}{a^4}\int_0^a\int_0^x\int_0^a\left\{\int_0^{z'} udz+\int_{z'}^a u_1dz\right\}dxdwdy.$$

$$=\frac{1}{2a^4}\int_0^a\int_0^x\int_0^a\left(\frac{2w^2y^2}{x}+a^2x-2awy\right)dxdwdy,$$